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A PRIMAL SIMPLEX CODE FOR COMPUTING THE EFFICIENCY OF DECISION --ETC(U)
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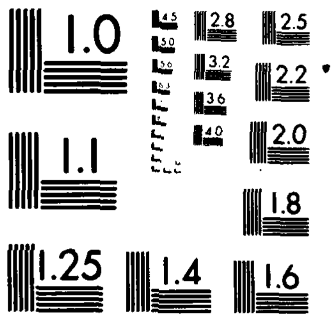

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(9) Research Report, CCS 383

(6) A PRIMAL SIMPLEX CODE FOR COMPUTING THE EFFICIENCY OF DECISION MAKING UNITS (VERSION 2.0),

by

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I. BASIC DESCRIPTION

The computer code (Version 2.0 of SMU1.0) described in this technical report was developed as part of research projects under the direction of Dr. A. Charnes (The University of Texas at Austin), Office of Naval Research Contract N00014-80-C-0242, and Dr. W.W. Cooper (The University of Texas at Austin), Office of Naval Research Contract N00014-76-C-0932. Dr. Authella Bessent (The University of Texas at Austin), Dr. Wailand Bessent (The University of Texas at Austin), and Dr. J. Kennington (Southern Methodist University) joined the group during 1979 and designed and tested SMU1.0 during the last six months of that year.

Given a set of decision making units (DMU's) with common inputs and outputs, SMU1.0 will compute the efficiency of each of the DMU's. The underlying theory is described in Charnes, Cooper, and Rhodes [1].

SMU1.0 is written in FORTRAN and was tested on SMU's CDC Cyber 73 using the MNF compiler. The code employs a modified primal simplex code which was originally developed by Iqbal Ali (The University of Texas at Austin) as a part of his dissertation work. The input format and output reports were designed by Authella and Wailand Bessent.

II. MODEL AND SOFTWARE DESIGN

To determine the efficiency of any DMU requires the solution of a particular linear program. In this section, we present the mathematical model, we give the initial feasible basis used to initiate the primal simplex algorithm, and we briefly discuss the design of the system.

A. Notation

m = number of DMU's

n_1 = number of outputs

n_2 = number of inputs

A = an $n_1 \times m$ matrix of outputs

C = an $n_2 \times m$ matrix of inputs

$A(j)$ = the j^{th} column of A

$C(j)$ = the j^{th} column of C

a_{ij} = the $(i, j)^{th}$ element of A

c_{ij} = the $(i, j)^{th}$ element of C

$\underline{e}_i$ = a column vector having i^{th} component equal +1 and all other components zero.

B. Assumptions

$a_{ij} \neq 0$ for all i, j.

$c_{ij} \neq 0$ for all i, j.

C. Model

Consider the following linear program:

$$
P(j) \left\{ \begin{array}{ll} \max & z \\ \text{s.t.} & -A(j)z + A\lambda + s_1 = 0 \\ & C\lambda + s_2 = C(j) \\ & z, \lambda, s_1, s_2 \geq 0 \end{array} \right\}
$$

where λ is an m vector, s_1 is an n_1 vector of slacks, and s_2 is an n_2 vector of slacks. Let $(z^*, \lambda^*, s_1^*, s_2^*)$ denote an optimum for P(j).

$$\text{If} \begin{cases} z^* = 1, \text{ then DMU } j \text{ is said to be efficient;} \\ z^* > 1, \text{ then DMU } j \text{ is said to be inefficient.} \end{cases}$$

D. Initial Feasible Basis

The following matrix is a feasible basis for P(j) with $z = \lambda_j = 1$ and all other variables equal zero:

$$\left[\begin{array}{c|c|c|c|c|c|c|c|c|c} -A(j) & A(j) & \underline{e}_2 & \underline{e}_3 & \cdots & \underline{e}_{n_1} & 0 & 0 & \cdots & 0 \\ \hline 0 & C(j) & 0 & 0 & \cdots & 0 & \underline{e}_2 & \underline{e}_3 & \cdots & \underline{e}_{n_2} \end{array}\right] \quad (1)$$

E. Software Design

SMU1.0 solves P(1) through P(m) via the revised primal simplex method using the starting bases (1) for each problem, P(j). The basis inverse is maintained in product form and only the non-zeroes are stored. The reinversion routine is based on the work of Hellerman and Rarick [4]. This routine incorporates a technique known as "splitting the bump" (see Kalan [5] and Orchard-Hays [6]) and uses the "spike swapping theory" of Helgason and Kennington [2]. A good description of the computational procedures may be found in Helgason [3]. The arrays and working files require approximately $(3m + 13)(n_1 + n_2) + 4m$ words of core storage.

III. INPUT FORMAT

Card Group	Source File	Composition	Description and Format
1	Tape 1	1 card	"Problem Title" (13A6, A2)
2	Tape 1	1 card	"Number of DMU's", "Number of Outputs", "Number of Inputs" (3I5)
3	Tape 1	card group	"Output 1 Label", "Output 2 Label", ..., "Output Last Label", "Input 1 Label", "Input 2 Label", ..., "Input Last Label" (8A10)
4	Tape 1	1 card	"Blank Card"
5	Tape 1	card group for each DMU	Outputs for the DMU followed by Inputs for the DMU (10F8.0)

Card groups 6 and 7 are optional. If included, each DMU's name will appear on its output report.

Card Group	Source File	Composition	Description and Format
6	Tape 1	1 card	"Blank Card" (Use only if card Group 7 follows)
7	Tape 1	card group for each DMU (in same order as DMU data)	"DMU1 name" (up to 80 characters) "DMU2 name" ⋮ "DMUm name"

Card group 8 is optional. A blank card defaults to an analysis of all DMU's.

Card Group	Source File	Composition	Description and Format
8	Input	1 card	"First DMU to be evaluated in this run", "Last DMU to be evaluated in this run" (free format, i.e., the two numbers may be in any columns as long as they are in the correct sequence and separated by either a blank or a comma)

IV. SET CODE DIMENSIONS

To set the dimension for a problem one should use a text editor which has the capability of replacing all occurrences of a specified character string by a number.

A. The Main Working Storage

Replace "ZMAT" with "----". $\left(\text{Should be at least } (m + n_1 + n_2 + 3)(n_1 + n_2 + 2) + (n_1 + n_2 + 2)^2\right)$

B. For The Number of Inefficient DMU's

Replace "ZINEFF" with "------". (m is sufficiently large)

C. For Column Length Arrays

Replace "ZCOL" with "----". (Should be at least $m + n_1 + n_2 + 3$)

D. For Row Length Arrays

Replace "ZROW" with "----". (Should be at least $n_1 + n_2 + 2$)

E. For The Length Of The ETA File

Replace "ZETA" with"----". (Should be at least 2m)

<u>WARNING</u>: For each of the above replacements, replace the words with a number having the same number of digits. Use leading zeroes if necessary. Suppose m = 100. Then replace ZETA with 0200 and replace ZINEFF with 000100.

SAMPLE SCHOOL DISTRICT ANALYSIS

```
COL =  100
ROW =   55
MAT = 4200
ETA =  120
INEFF =   00
  27    5    7

RUN FOR DMU   1 THRU DMU  10

RUN FOR DMU   1 THRU DMU  10

 NCE READ    NCE ENG     NCE MATH    PROF/100    $/PUPIL     $NON-MIN    %PAY LUNCH  %EXP.TEACH  STU ATTEND  TEACH.ATT
```

SAMPLE SCHOOL DISTRICT ANALYSIS

DECISION MAKING UNIT 1 SCHOOL 1 NAME

EFFICIENCY = 1.000

INEFFICIENCY = 1.000

	VALUE MEASURED	VALUE IF EFFICIENT	SLACK	DUAL	RIGHT-HAND SIDE RANGES MINIMUM VALUE	RIGHT-HAND SIDE RANGES MAXIMUM VALUE
*********** * OUTPUTS * ***********						
NCE READ	56.1000	56.1000	0.0000	.0100	-4.4046	0.0000
NCE ENG	57.4000	57.4000	0.0000	0.0000		
NCE MATH	54.7000	54.7000	0.0000	.0080	0.0000	0.0000
*********** * INPUTS * ***********						
PROF/100	6.3000	6.3000	0.0000	0.0000		
$/PUPIL	970.4100	970.4100	0.0000	0.0000		
%NON-MIN	87.0000	87.0000	0.0000	0.0000		
%PAY LUNCH	95.0000	95.0000	0.0000	0.0000		
%EXP.TEACH	39.0000	39.0000	0.0000	.0250	0.0000	39.0000
$IU ATTEND	98900.0000	98900.0000	0.0000	0.0000		
TEACH.ATT	96.2000	96.2000	0.0000	0.0000		

SAMPLE SCHOOL DISTRICT ANALYSIS

DECISION MAKING UNIT 2 SCHOOL 2 NAME

EFFICIENCY = .870

INEFFICIENCY = 1.149

	VALUE MEASURED	VALUE IF EFFICIENT	SLACK	DUAL	RIGHT-HAND SIDE RANGES MINIMUM VALUE	RIGHT-HAND SIDE RANGES MAXIMUM VALUE
* OUTPUTS *						
NCE READ	50.5000	68.5993	10.3560	0.0000		
NCE ENG	55.3000	67.4593	3.6990	0.0000		
NCE MATH	54.2000	62.2960	0.0000	.0185	-62.2960	3.8215
* INPUTS *						
PROF/100	5.3000	5.2210	.0790	0.0000		
$/PUPIL	1614.0000	959.6402	654.3598	0.0000		
%NON-MIN	7.3000	7.3000	0.0000	.0065	.1018	22.3090
%PAY LUNCH	91.0000	70.5582	20.4418	0.0000		
%EXP.TEACH	64.0000	48.7943	15.6057	0.0000		
STU ATTEND	91.4000	90.3192	1.0808	0.0000		
TEACH.ATT	95.2000	95.2000	0.0000	.0120	13.6605	90.3458

SAMPLE SCHOOL DISTRICT ANALYSIS

DECISION MAKING UNIT 10 SCHOOL 10 NAME

EFFICIENCY = .726

INEFFICIENCY = 1.378

	VALUE MEASURED	VALUE IF EFFICIENT	SLACK	DUAL	RIGHT-HAND SIDE RANGES MINIMUM VALUE	MAXIMUM VALUE
*********** * OUTPUTS * ***********						
NCE READ	48.4000	69.0524	2.3695	0.0000		
NCE ENG	49.5000	68.1984	0.0000	.0062	-1.5412	.8712
NCE MATH	46.3000	63.7896	0.0000	.0150	-.8149	1.4415
*********** * INPUTS * ***********						
PROF/100	7.5000	5.3630	2.1370	0.0000		
$/PUPIL	2062.0000	1077.1072	984.8928	0.0000		
$NON-MIN	46.4000	17.0866	29.3134	0.0000		
$PAY LUNCH	95.6000	75.9187	19.6813	0.0000		
$EXP.TEACH	67.6000	53.0724	14.5276	0.0000		
STU ATTEND	93.5000	92.8124	.6876	0.0000		
TEACH.ATT	97.1000	97.1000	0.0000	.0142	0.0000	97.8194

EFFICIENCY/INEFFICIENCY SUMMARY

DMU	H0	Z0
1	1.00000	1.00000
2	.87004	1.14937
3	1.00000	1.00000
4	.54909	1.82118
5	.70090	1.42674
6	.58183	1.71872
7	.53673	1.86313
8	.96925	1.03175
9	.75425	1.32582
10	.72582	1.37775

SUMMARY OF SLACK VALUES--INEFFICIENT UNITS ONLY

OUTPUT/INPUT	2	4	5	6	7	8
NCE READ	10.35598035	4.62100805	8.01561497	7.55153577	7.52094972	3.95527235
NCE ENG	5.89902092	0.00000000	0.00000000	0.00000000	0.00000000	5.02645479
NCE MATH	0.00000000	.69552544	1.15800214	4.32410043	3.80195531	0.00000000
PROF/100	.07901001	2.21734694	.47454545	.99591837	.50000000	0.00000000
$/PUPIL	654.35976079	1230.55057823	762.17888492	857.82653061	746.59000000	473.04425117
%NON-MIN	0.00000000	2.00034014	48.79754011	4.29795918	0.00000000	63.72840130
%PAY LUNCH	20.44100372	15.72585034	27.80663102	28.04489796	22.62000000	12.58422952
%EXP.TEACH	15.80569608	26.05476190	19.08074866	9.37142857	4.50000000	19.11706965
STU ATTEND	1.00877874	0.00000000	.63037433	0.00000000	4.30000000	2.35976182
TEACH.ATT	0.00000000	1.71002721	0.00000000	.29183673	1.20000000	0.00000000

SUMMARY OF SLACK VALUES--INEFFICIENT UNITS ONLY

OUTPUT/INPUT	9	10
NCE READ	9.51088036	2.36949816
NCE ENG	0.00000000	0.00000000
NCE MATH	1.84342039	0.00000000
PROF/100	.66909091	2.13697149
$/PUPIL	772.24792513	984.89283046
%NON-MIN	23.09743316	29.31342350
%PAY LUNCH	24.19561497	19.68128563
%EXP.TEACH	18.23208556	14.52764251
STU ATTEND	4.53604278	.08764075
TEACH.ATT	0.00000000	0.00000000

SUMMARY OF DUAL VALUES--INEFFICIENT UNITS ONLY

OUTPUT/INPUT	2	4	5	6	7	8
NCE READ	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
NCE ENG	0.00000000	.02759726	.02087665	.02525253	.02743246	0.00000000
NCE MATH	.01045010	0.00000000	0.00000000	0.00000000	0.00000000	.01639344
PROF/TAD	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	.10844165
$/PUPIL	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
$NON-MIN	.00047909	0.00000000	0.00000000	0.00000000	10.63128492	0.00000000
$PAT LUNCH	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
$EXP.TEACH	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
STU ATTEND	0.00000000	.02071879	0.00000000	.01904686	0.00000000	0.00000000
TEACH.ATT	.01203051	0.00000000	.01489280	0.00000000	0.00000000	.00478019

SUMMARY OF DUAL VALUES--INEFFICIENT UNITS ONLY

OUTPUT/INPUT	9	10
NCE READ	0.00000000	0.00000000
NCE ENG	.01937984	.00018010
NCE MATH	0.00000000	.01499104
PROF/100	0.00000000	0.00000000
$/PUPIL	0.00000000	0.00000000
%NON-MIN	0.00000000	0.00000000
%PAY LUNCH	0.00000000	0.00000000
%EXP.TEACH	0.00000000	0.00000000
STU ATTEND	0.00000000	0.00000000
TEACH.ATT	.01382498	.01418894

REFERENCES

[1] Charnes, A., W. W. Cooper, and E. Rhodes, "Measuring the Efficiency of Decision Making Units", European Journal of Operations Research, 2, (1978) 429-444.

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19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

decision making units
efficiency
primal simplex method
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"splitting the bump"
"spike swapping theory"

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This technical report describes the computer code (Version 2.0 of SMU1) developed to compute the efficiency of a set of decision making units (DMU's) with common inputs and outputs. The code, written in FORTRAN and tested on a CDC Cyber 73 using an MNF compiler, employs a modified primal simplex code. A sample of the computer generated solution is included.